

Interpolation

HES 505 Fall 2023: Session 19

Matt Williamson

Objectives

By the end of today you should be able to:

- Distinguish deterministic and stochastic processes
- Define autocorrelation and describe its estimation
- Articulate the benefits and drawbacks of autocorrelation
- Leverage point patterns and autocorrelation to interpolate missing data

But first...

Patterns as realizations of spatial processes

- A **spatial process** is a description of how a spatial pattern might be *generated*
- **Generative models**
- An observed pattern as a *possible realization* of an hypothesized process

Deterministic vs. stochastic processes

- Deterministic processes: always produce the same outcome

$$z = 2x + 3y$$

- Results in a spatially continuous field

Deterministic vs. stochastic processes

```
1 x <- rast(nrows = 10, ncols=10, xmin = 0, xmax=10, ymin = 0, ymax=10)
2 values(x) <- 1
3 z <- x
4 values(z) <- 2 * crds(x)[,1] + 3*crds(x)[,2]
```

Deterministic vs. stochastic processes

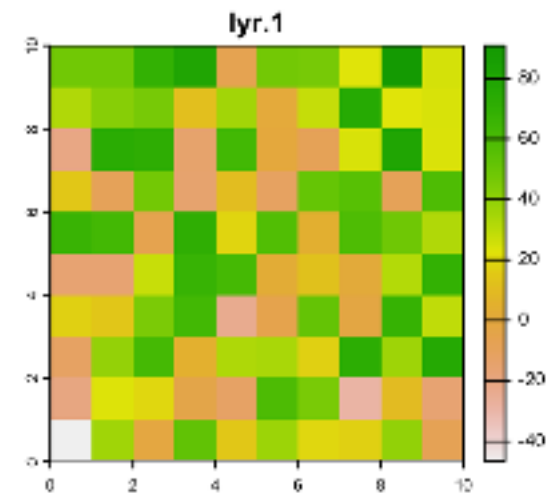
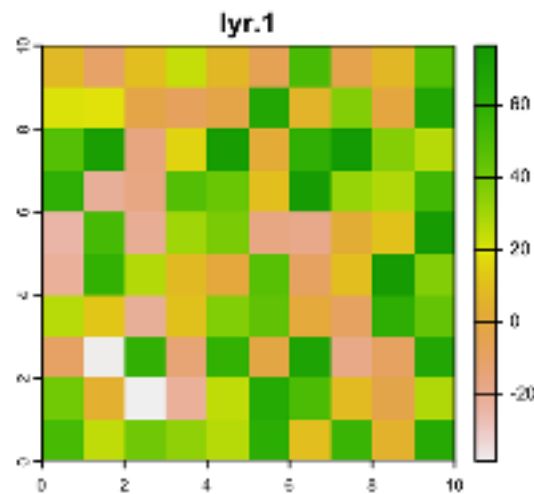
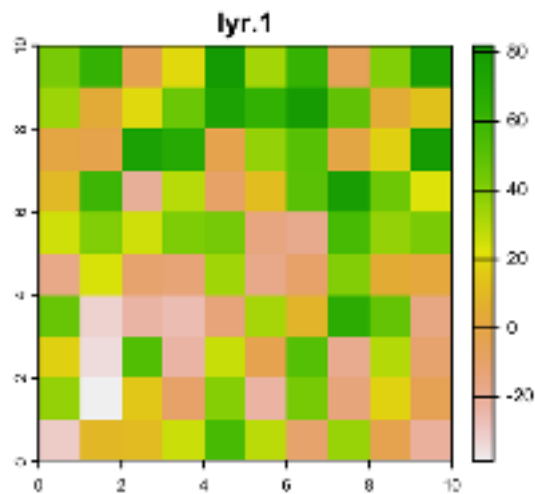
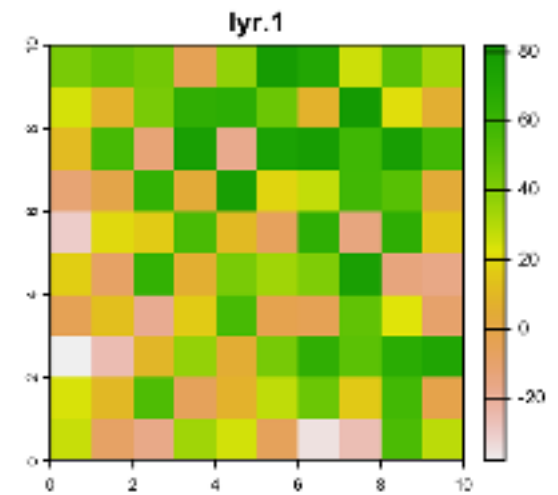
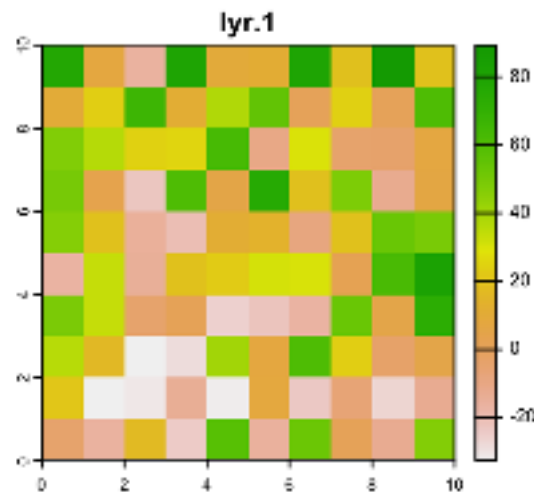
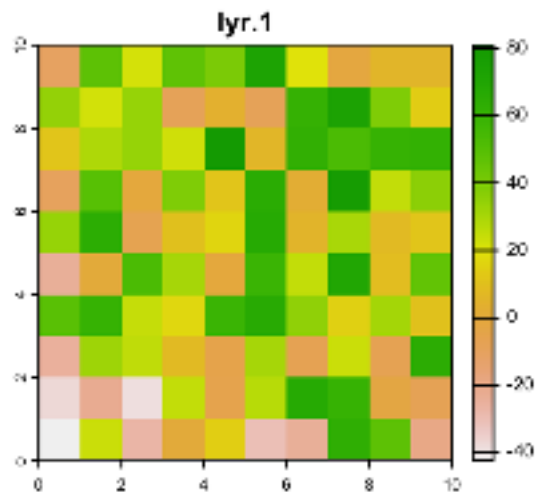
- Stochastic processes: variation makes each realization difficult to predict

$$z = 2x + 3y + d$$

- The *process* is random, not the result (!!)
- Measurement error makes deterministic processes appear stochastic

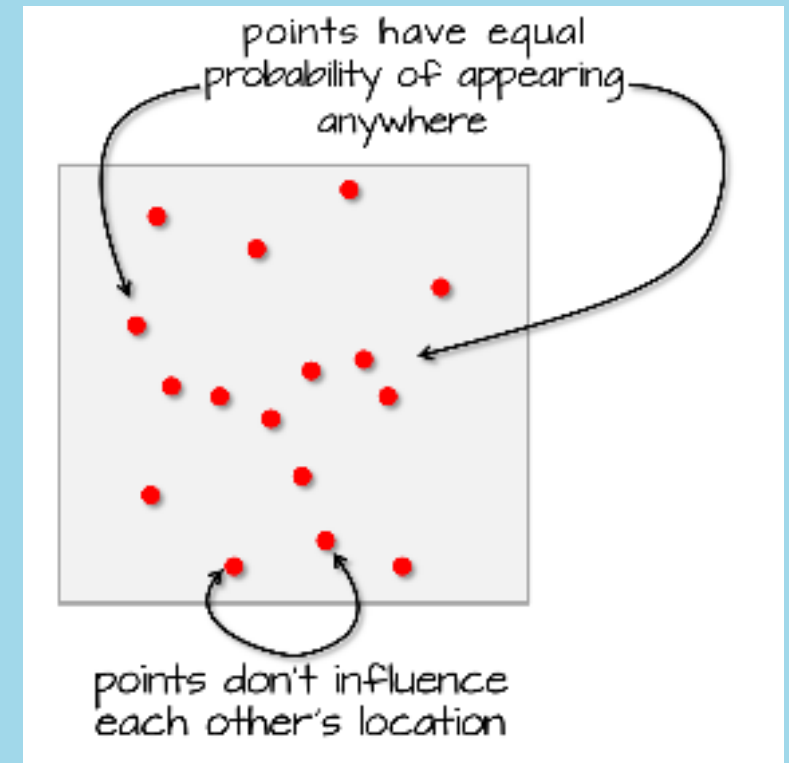
```
1 x <- rast(nrows = 10, ncols=10, xmin = 0,
2 values(x) <- 1
3 fun <- function(z){
4 a <- z
5 d <- runif(ncell(z), -50, 50)
6 values(a) <- 2 * crds(x)[,1] + 3*crds(x)[,
7 return(a)
8 }
9
10 b <- replicate(n=6, fun(z=x), simplify=FAI
11 d <- do.call(c, b)
```

Deterministic vs. stochastic processes



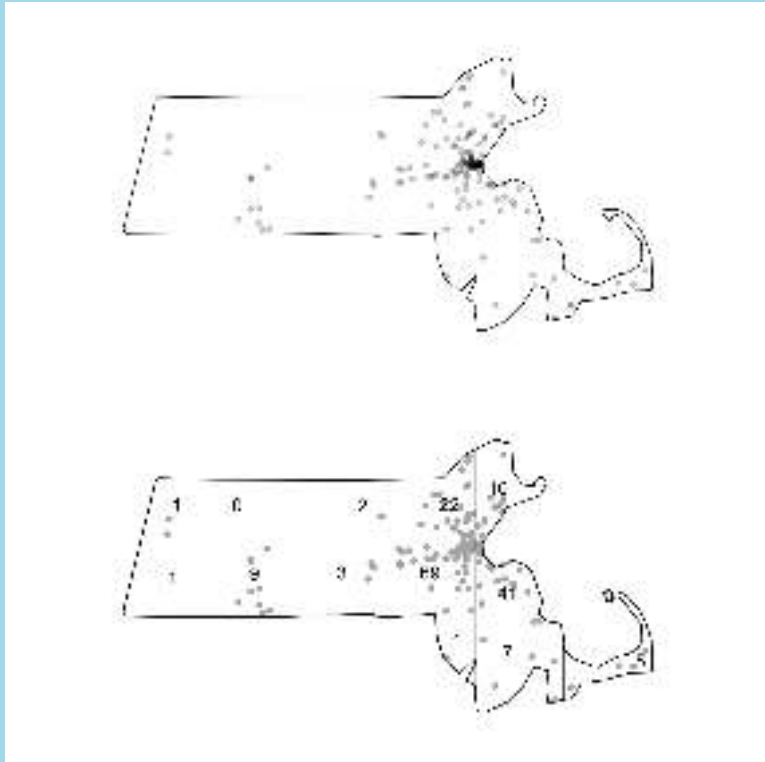
Expected values and hypothesis testing

- Considering each outcome as the realization of a process allows us to generate expected values
- The simplest spatial process is Completely Spatial Random (CSR) process
- **First Order** effects: any event has an equal probability of occurring in a location
- **Second Order** effects: the location of one event is independent of the other events



From Manuel Gimond

Generating expactations for CSR



- We can use quadrat counts to estimate the expected number of events in a given area
- The probability of each possible count is given by:

$$P(n, k) = \binom{n}{x} p^k (1 - p)^{n-k}$$

- Given total coverage of quadrats, then $p = \frac{\frac{a}{x}}{a}$ and

$$P(k, n, x) = \binom{n}{k} \left(\frac{1}{x} \right)^k \left(\frac{x-1}{x} \right)^{n-k}$$

Revisiting Ripley's K

- Nearest neighbor methods throw away a lot of information
- If points have independent, fixed marginal densities, then they exhibit *complete, spatial randomness* (CSR)
- The K function is an alternative, based on a series of circles with increasing radius

$$K(d) = \lambda^{-1} E(N_d)$$

- We can test for clustering by comparing to the expectation:

$$K_{\text{CSR}}(d) = \pi d^2$$

- if $k(d) > K_{\text{CSR}}(d)$ then there is clustering at the scale defined by d

Ripley's K Function

- When working with a sample the distribution of K is unknown
- Estimate with

$$\hat{K}(d) = \hat{\lambda}^{-1} \sum_{i=1}^n \sum_{j=1}^n \frac{I(d_{ij} < d)}{n(n-1)}$$

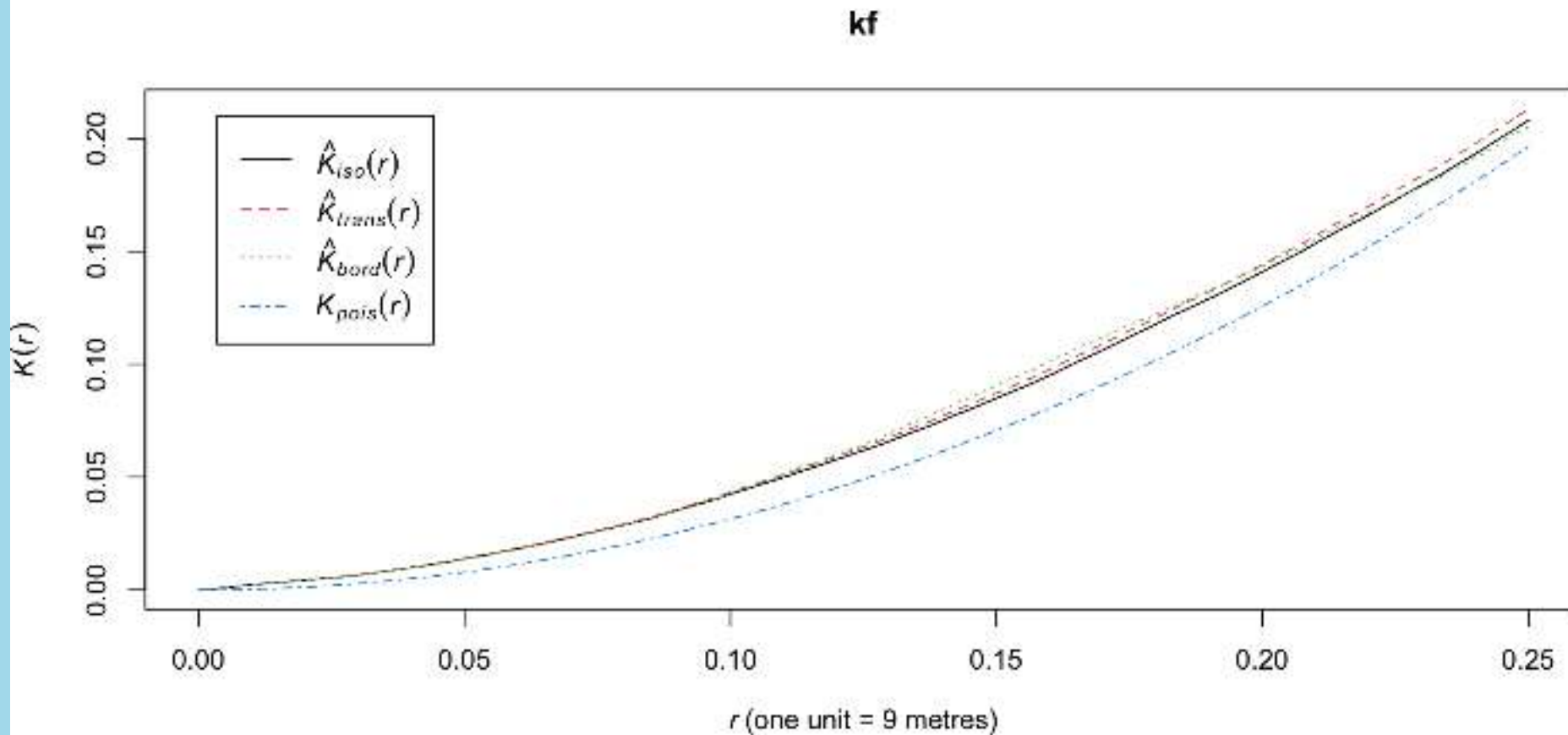
where:

$$\hat{\lambda} = \frac{n}{|A|}$$

Ripley's K Function

Ripley's K Function

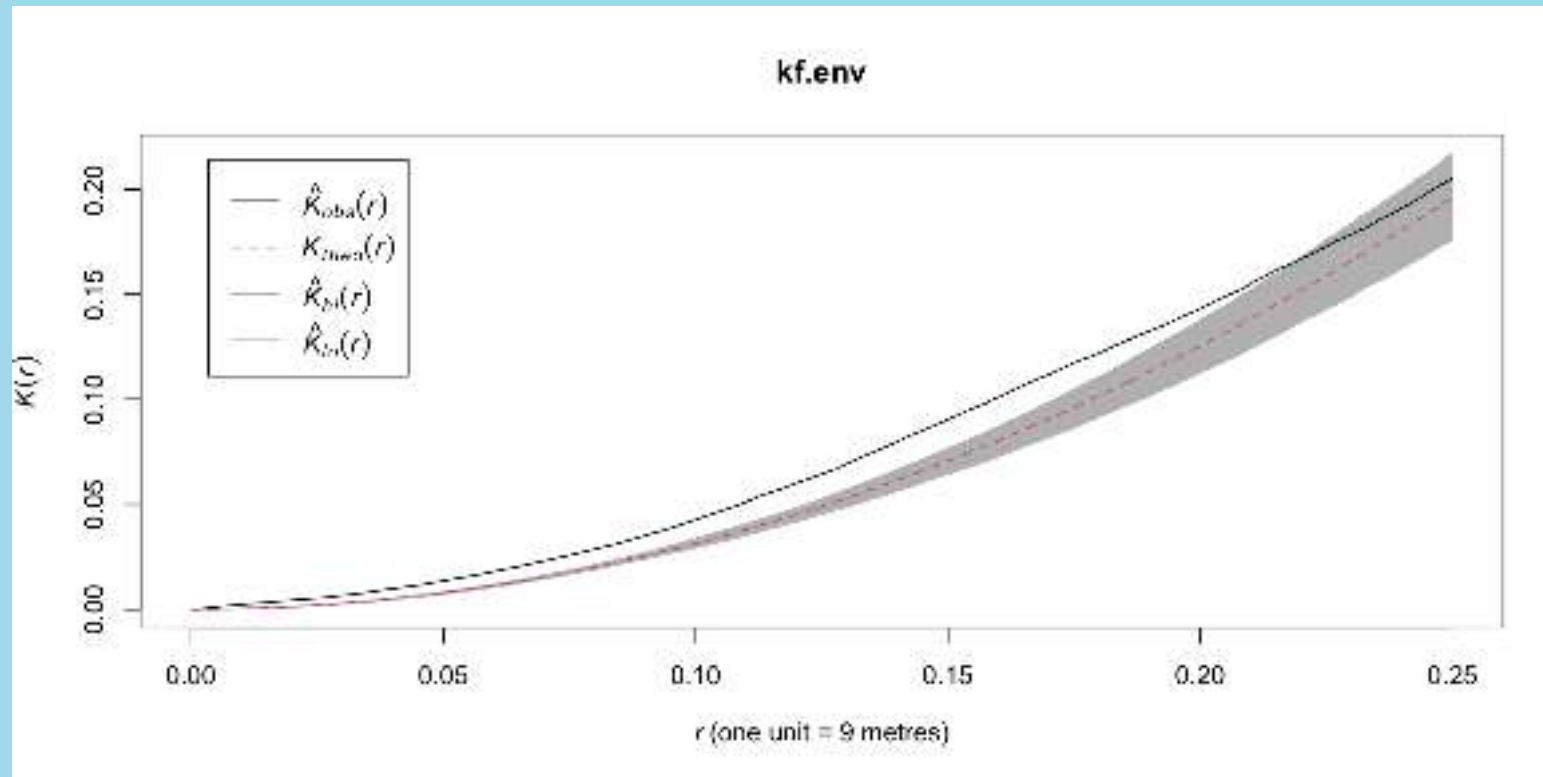
```
1 kf <- Kest(bramblecanes, correction="border")  
2 plot(kf)
```



Ripley's K Function

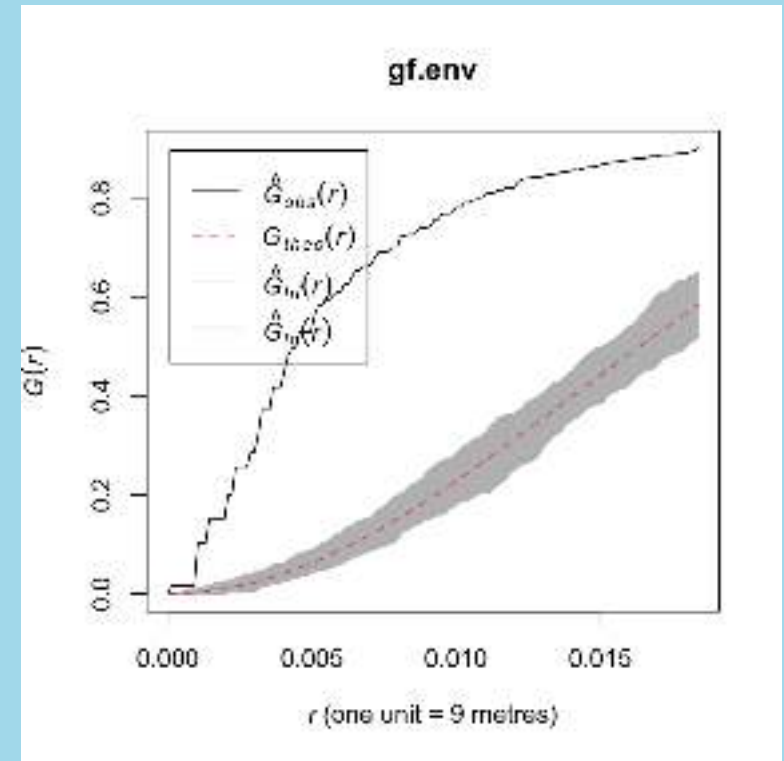
- accounting for variation in d

```
1 kf.env <- envelope(bramblecanes, correction="border", envelope = FALSE, ver  
2 plot(kf.env)
```



Other functions

- L function: square root transformation of K
- G function: the cumulative frequency distribution of the nearest neighbor distances
- F function: similar to G but based on randomly located points

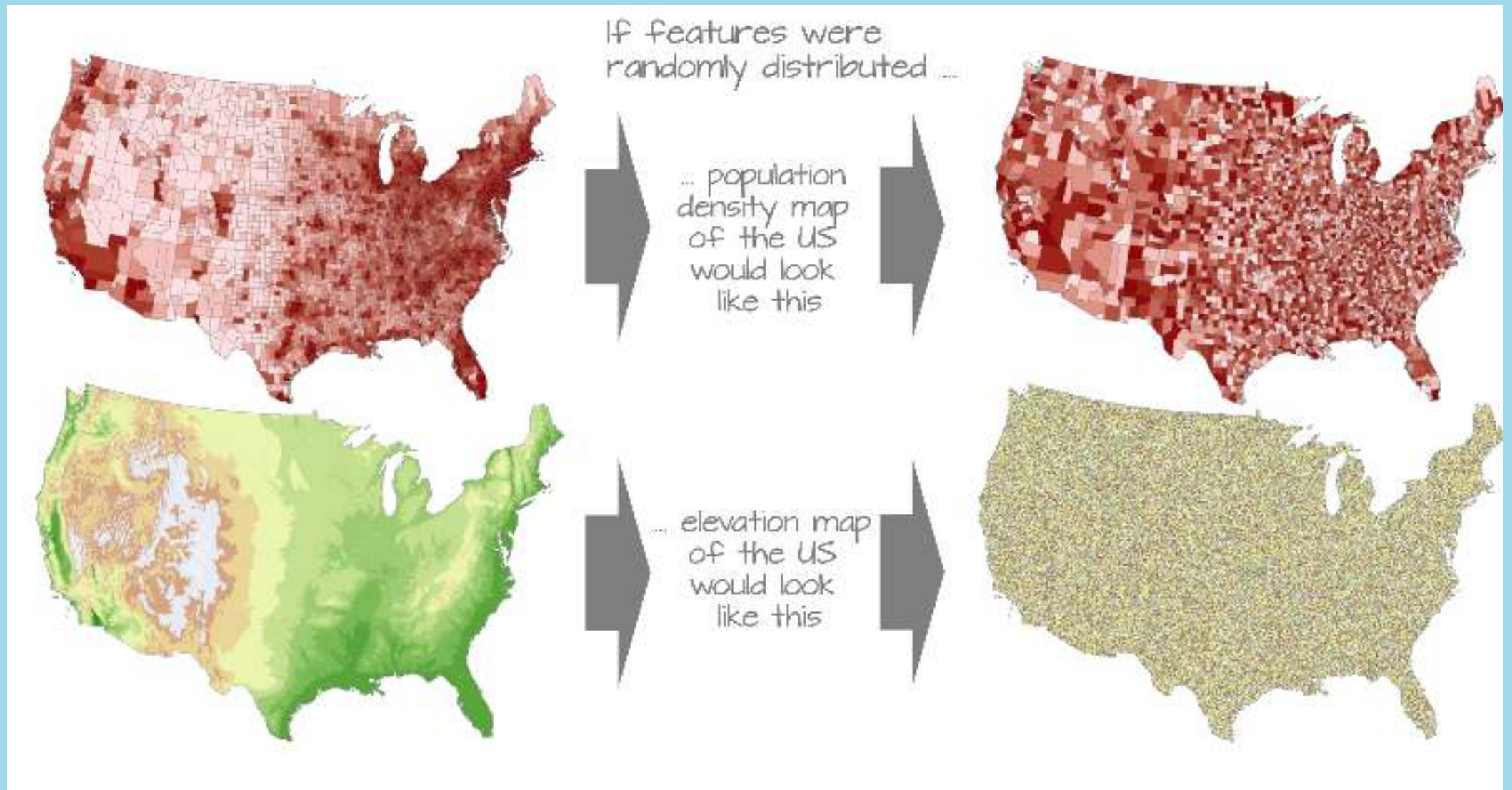


Tobler's Law

‘everything is usually related to all else but those which are near to each other are more related when compared to those that are further away’.

Waldo Tobler

Spatial autocorrelation

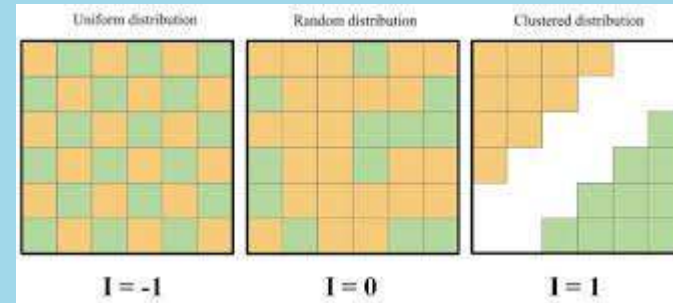


From Manuel Gimond

(One) Measure of autocorrelation

- Moran's I

$$I(d) = \frac{\sum_i \sum_{j \neq i} w_{ij} (x_i - \bar{x})(x_j - \bar{x})}{S^2 \sum_i \sum_{j \neq i} w_{ij}}$$



Moran's I: An example

- Use **spdep** package
- Estimate neighbors
- Generate weighted average

```
1 set.seed(2354)
2 # Load the shapefile
3 s <- readRDS(url("https://github.com/mgimond/Data/raw/gh-pages"))
4
5 # Define the neighbors (use queen case)
6 nb <- poly2nb(s, queen=TRUE)
7
8 # Compute the neighboring average homicide rates
9 lw <- nb2listw(nb, style="W", zero.policy=TRUE)
10 #estimate Moran's I
11 moran.test(s$HR80,lw, alternative="greater")
```

Moran I test under randomisation

data: s\$HR80

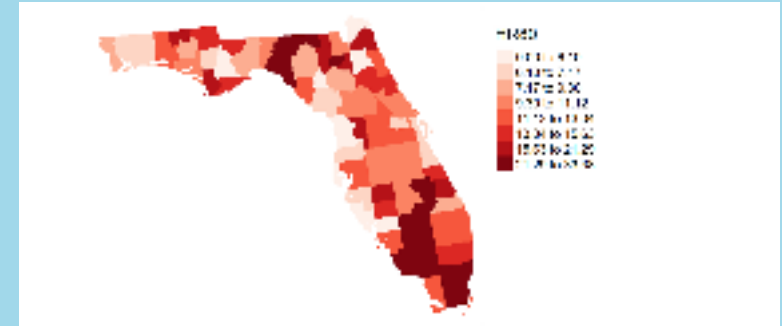
weights: lw

Moran I statistic standard deviate = 1.8891, p-value = 0.02944

alternative hypothesis: greater

sample estimates:

Moran I statistic	Expectation	Variance
0.136277593	-0.015151515	0.006425761



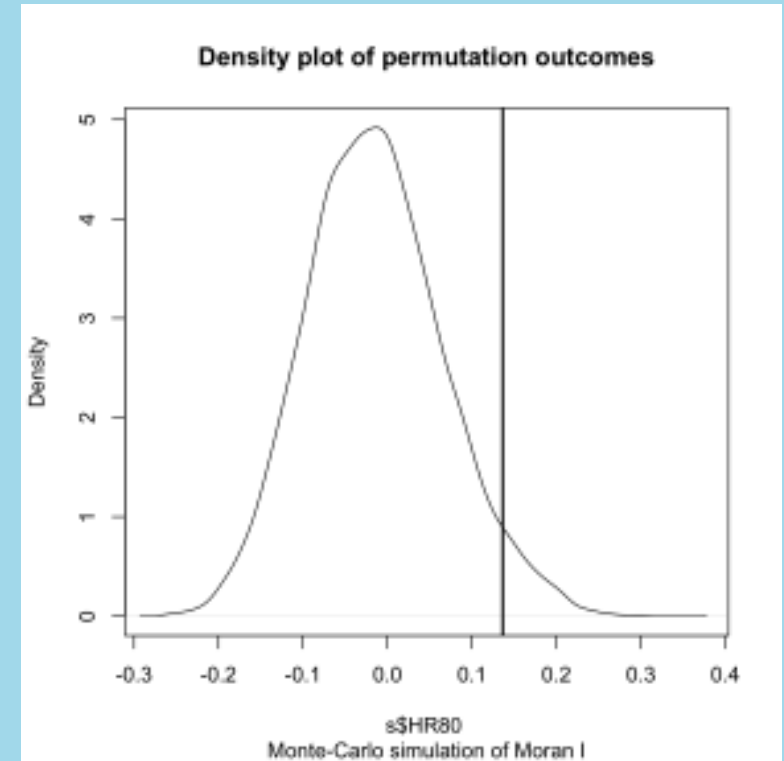
Moran's I: An example

```
1 M1 <- moran.mc(s$HR80, lw, nsim=9999, alte  
2  
3  
4  
5 # Display the resulting statistics  
6 M1
```

Monte-Carlo simulation of Moran I

data: s\$HR80
weights: lw
number of simulations + 1: 10000

statistic = 0.13628, observed rank = 9575, p-
value = 0.0425
alternative hypothesis: greater



The challenge of areal data

- Spatial autocorrelation threatens *second order* randomness
- Areal data means an infinite number of potential distances
- Neighbor matrices, W , allow different characterizations

Interpolation

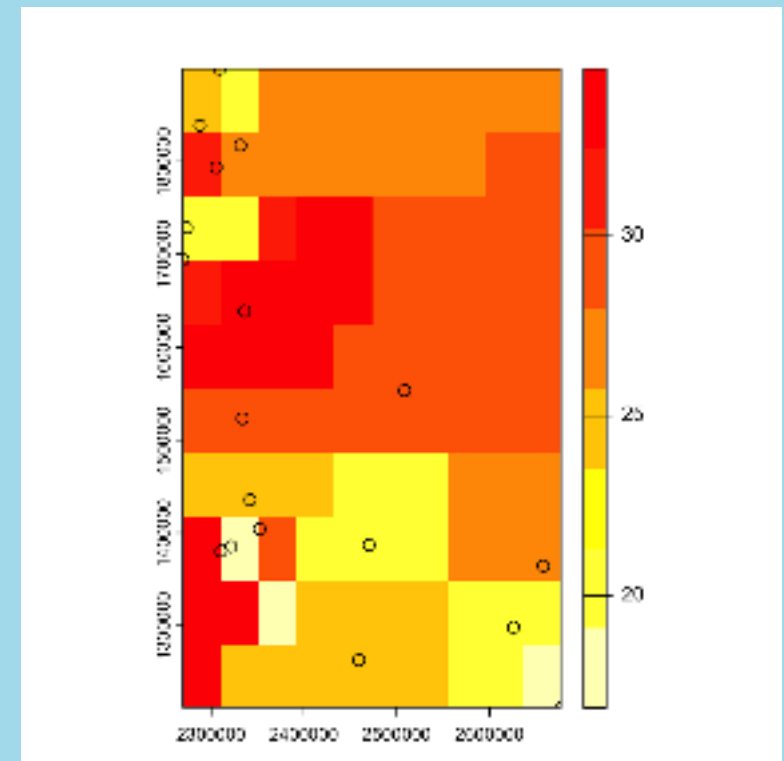
Interpolation

- Goal: estimate the value of z at new points in \mathbf{x}_i
- Most useful for continuous values
- Nearest-neighbor, Inverse Distance Weighting, Kriging

Nearest neighbor

- find i such that $|\mathbf{x}_i - \mathbf{x}|$ is minimized
- The estimate of z is z_i

```
1 aq <- read_csv("data/ad_viz_plotval_data.csv") %>%
2   st_as_sf(., coords = c("SITE_LONGITUDE", "SITE_LATITUDE"), crs = "EPSG:8826") %>%
3   st_transform(., crs = "EPSG:8826") %>%
4   mutate(date = as_date(parse_datetime(Date, "%m/%d/%Y"))) %>%
5   filter(., date >= 2023-07-01) %>%
6   filter(., date > "2023-07-01" & date < "2023-07-31")
7 aq.sum <- aq %>%
8   group_by(., `Site Name`) %>%
9   summarise(., meanpm25 = mean(DAILY_AQI_VALUE))
10
11 nodes <- st_make_grid(aq.sum,
12                       what = "centers")
13
14 dist <- distance(vect(nodes), vect(aq.sum))
15 nearest <- apply(dist, 1, function(x) which(x == min(x)))
16 aq.nn <- aq.sum$meanpm25[nearest]
17 preds <- st_as_sf(nodes)
18 preds$aq <- aq.nn
19
20 preds <- as(preds, "Spatial")
21 sp::gridded(preds) <- TRUE
22 preds.rast <- rast(preds)
```



Inverse-Distance Weighting

- Weight closer observations more heavily

$$\hat{z}(\mathbf{x}) = \frac{\sum_{i=1} w_i z_i}{\sum_{i=1} w_i}$$

where

$$w_i = |\mathbf{x} - \mathbf{x}_i|^{-\alpha}$$

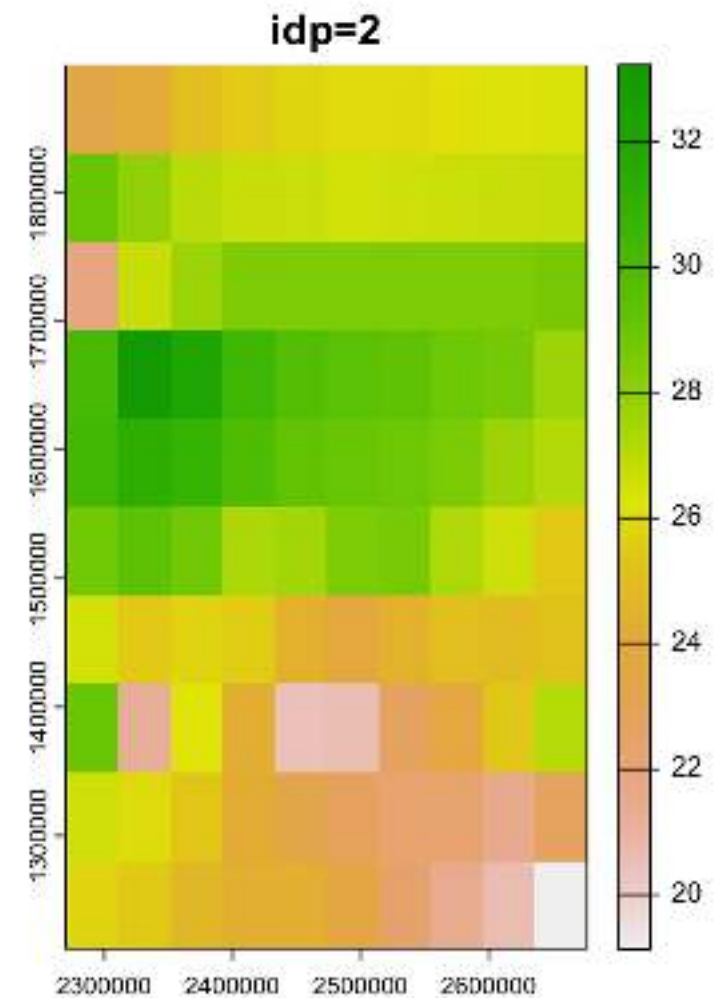
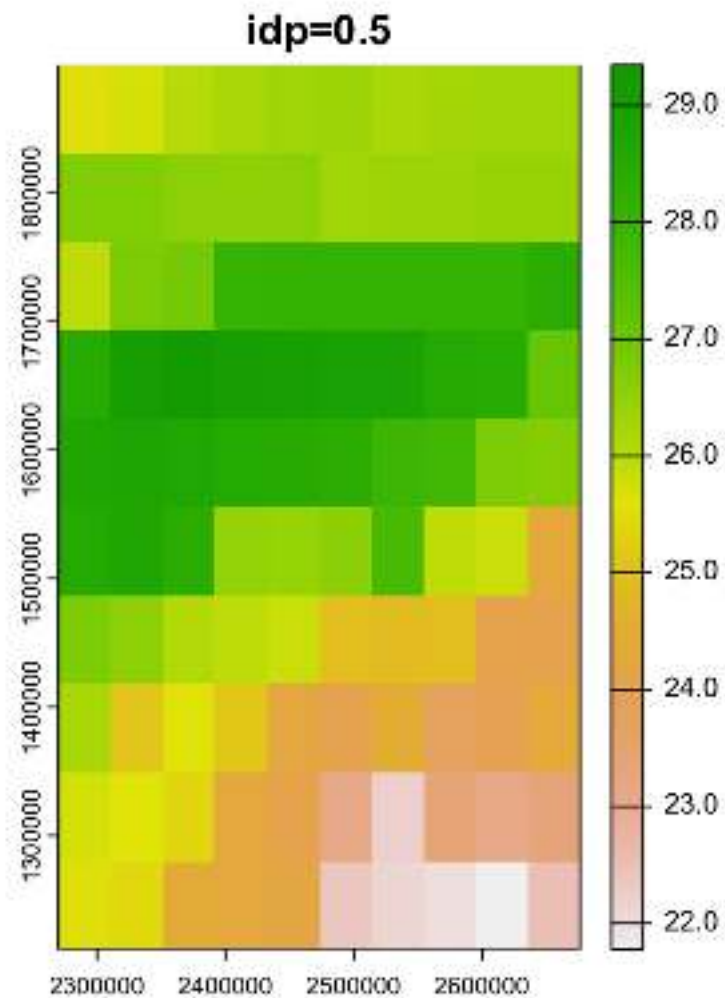
and $\alpha > 0$ ($\alpha = 1$ is inverse; $\alpha = 2$ is inverse square)

Inverse-Distance Weighting

- `terra::interpolate` provides flexible interpolation methods
- Use the `gstat` package to develop the formula

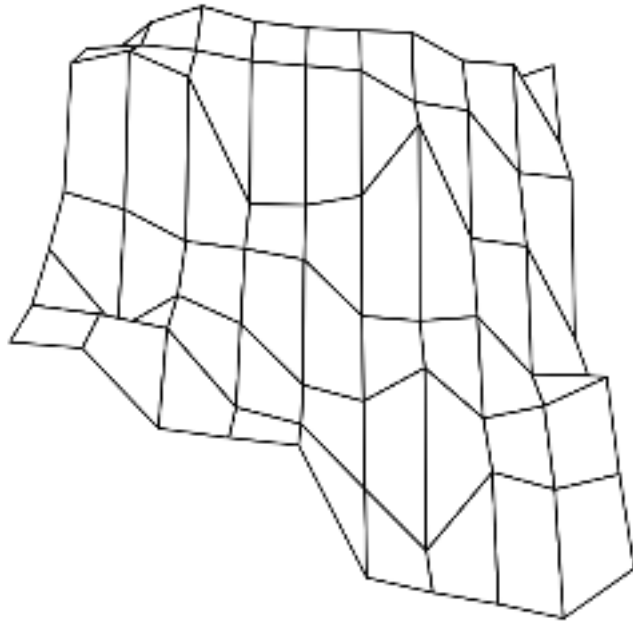
```
1 mgsf05 <- gstat(id = "meanpm25", formula = meanpm25~1, data=aq.sum, nmax=7)
2 mgsf2 <- gstat(id = "meanpm25", formula = meanpm25~1, data=aq.sum, nmax=7,
3 interpolate_gstat <- function(model, x, crs, ...) {
4     v <- st_as_sf(x, coords=c("x", "y"), crs=crs)
5     p <- predict(model, v, ...)
6     as.data.frame(p)[,1:2]
7 }
8 zsf05 <- interpolate(preds.rast, mgsf05, debug.level=0, fun=interpolate_gstat)
9 zsf2 <- interpolate(preds.rast, mgsf2, debug.level=0, fun=interpolate_gstat)
```

Inverse-Distance Weighting



Inverse-Distance Weighting

idp=0.5



idp=2

