# Statistical Modelling II

HES 505 Fall 2023: Session 23

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# Objectives

By the end of today you should be able to:

- Articulate the differences between statisitical learning classifiers and logistic regression
- Describe several classification trees and their relationship to Random Forests
- Describe MaxEnt models for presence-only data

# Revisiting Classification

# Favorability in General

$$F(\mathbf{s}) = f(w_1 X_1(\mathbf{s}), w_2 X_2(\mathbf{s}), w_3 X_3(\mathbf{s}), \ldots, w_m X_m(\mathbf{s}))$$

- Logistic regression treats $f(x)$ as a (generalized) linear function
- Allows for multiple qualitative classes
- Ensures that estimates of $F(\mathbf{s})$ are [0,1]

# Key assumptions of logistic regression

- Dependent variable must be binary
- Observations must be independent (important for spatial analyses)
- Predictors should not be collinear
- Predictors should be linearly related to the log-odds
- **Sample Size**

# Beyond Linearity

- Logistic (and other generalized linear models) are relatively interpretable
- Probability theory allows robust inference of effects
- Predictive power can be low
- Relaxing the linearity assumption can help

# Classification Trees

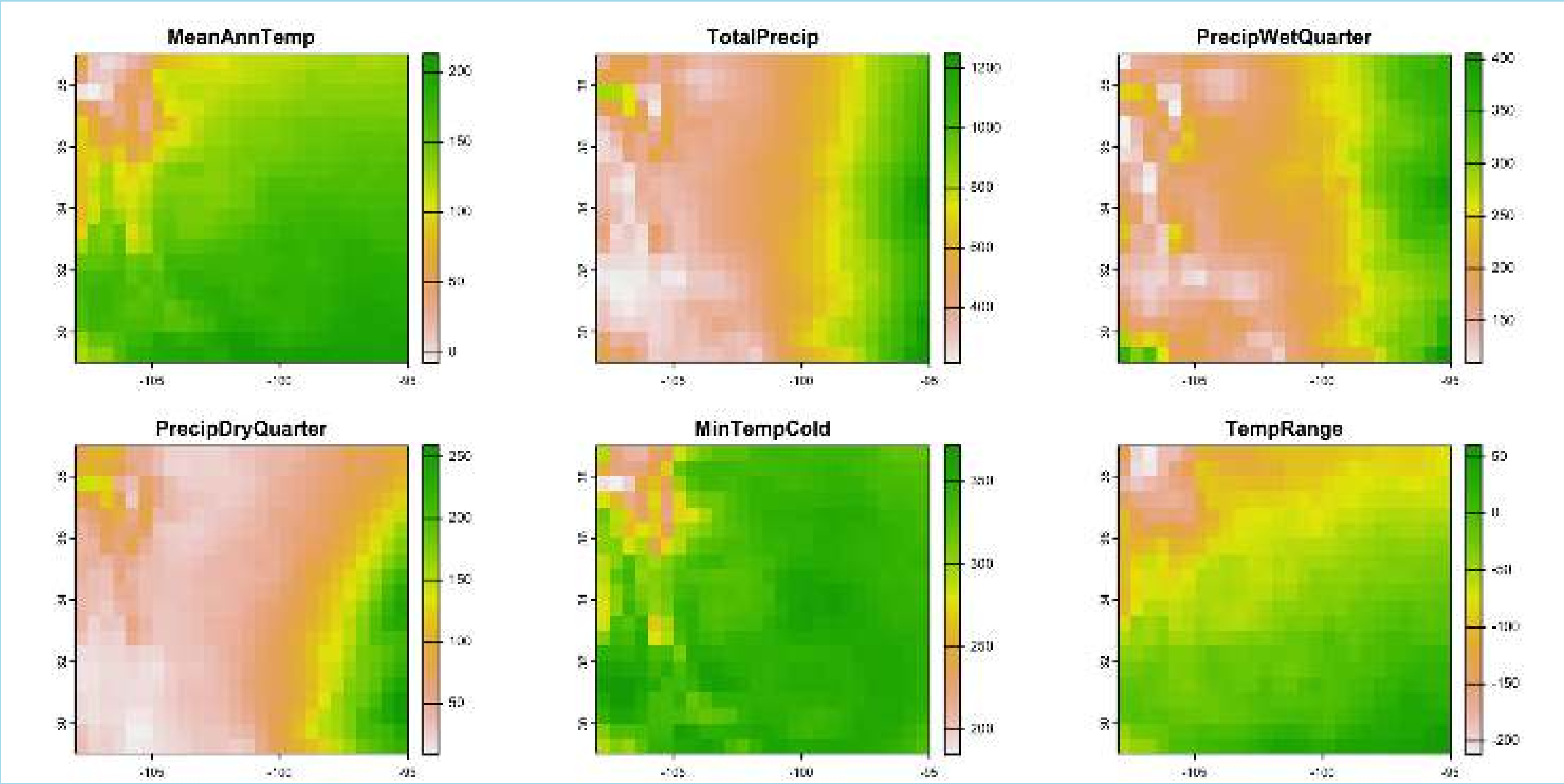
- Use decision rules to segment the predictor space
- Series of consecutive decision rules form a 'tree'
- Terminal nodes (leaves) are the outcome; internal nodes (branches) the splits

# Classification Trees

- Divide the predictor space (R) into J non-overlapping regions
- Every observation in $R_j$ gets the same prediction
- *Recursive binary splitting*
- Pruning and over-fitting

# An Example

## Inputs from the `dismo` package

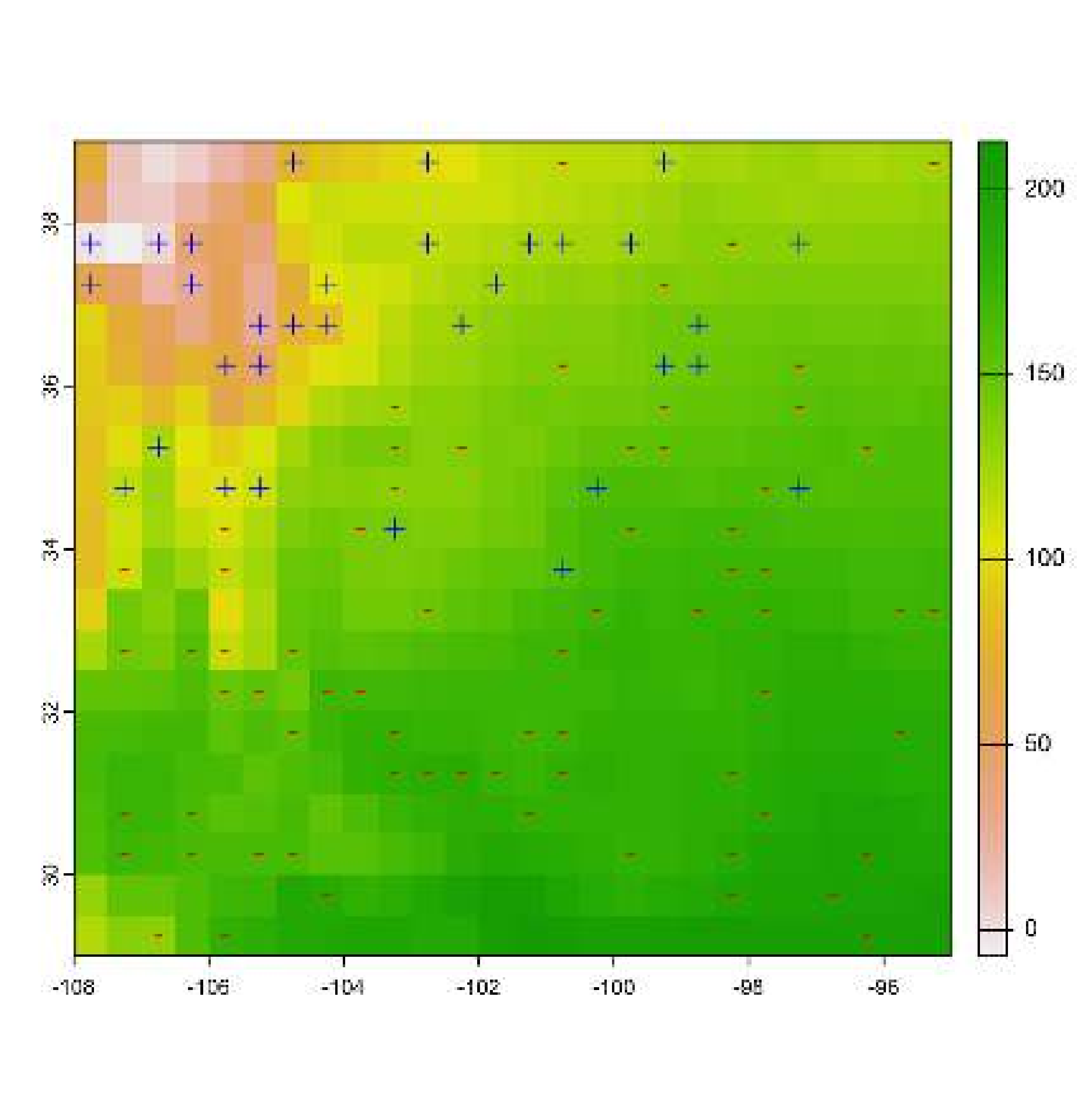


# An Example

The sample data

```r
head(pres.abs)
```

```
Simple feature collection
with 6 features and 1 field
Geometry type: POINT
Dimension:     XY
Bounding box:  xmin: -106.75
ymin: 31.25 xmax: -98.75
ymax: 37.75
Geodetic CRS:  GCS_unknown
  y              geometry
1 0  POINT (-99.25 35.25)
2 1  POINT (-98.75 36.25)
3 1 POINT (-106.75 35.25)
4 0 POINT (-100.75 31.25)
5 1  POINT (-99.75 37.75)
6 1 POINT (-104.25 36.75)
```



# An Example

## Building our dataframe

```r
pts.df <- terra::extract(pred.stack, vect(pres.abs), df=TRUE)
head(pts.df)
```

```
  ID MeanAnnTemp TotalPrecip PrecipWetQuarter PrecipDryQuarter MinTempCold
1  1         155         667              253               71         350
2  2         147         678              266               66         351
3  3         123         261              117               40         329
4  4         181         533              198               69         348
5  5         127         589              257               48         338
6  6          83         438              213               38         278
  TempRange
1       -45
2       -58
3       -64
4        -5
5       -81
6      -107
```

# An Example

## Building our dataframe

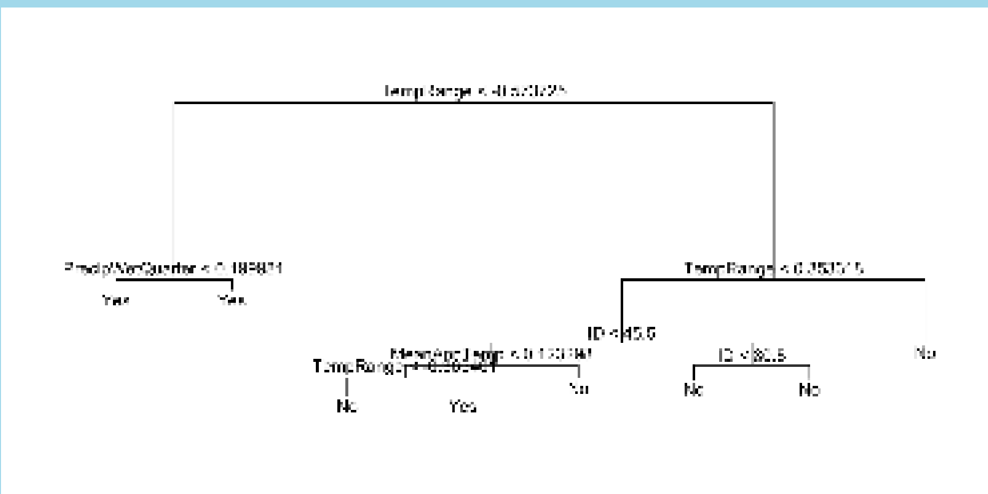
```r
pts.df[,2:7] <- scale(pts.df[,2:7])
summary(pts.df)
```

```
       ID           MeanAnnTemp        TotalPrecip       PrecipWetQuarter
 Min.   :  1.00   Min.   :-3.3729   Min.   :-1.3377   Min.   :-1.6926
 1st Qu.: 25.75   1st Qu.:-0.4594   1st Qu.:-0.7980   1st Qu.:-0.6895
 Median : 50.50   Median : 0.2282   Median :-0.2373   Median :-0.2224
 Mean   : 50.50   Mean   : 0.0000   Mean   : 0.0000   Mean   : 0.0000
 3rd Qu.: 75.25   3rd Qu.: 0.7118   3rd Qu.: 0.7140   3rd Qu.: 0.6508
 Max.   :100.00   Max.   : 1.4285   Max.   : 2.4843   Max.   : 2.2713
 PrecipDryQuarter   MinTempCold         TempRange
 Min.   :-1.0828   Min.   :-3.9919   Min.   :-2.7924
 1st Qu.:-0.7013   1st Qu.:-0.0598   1st Qu.:-0.5216
 Median :-0.3770   Median : 0.3582   Median : 0.2075
 Mean   : 0.0000   Mean   : 0.0000   Mean   : 0.0000
 3rd Qu.: 0.4290   3rd Qu.: 0.5495   3rd Qu.: 0.6450
 Max.   : 3.1713   Max.   : 1.1092   Max.   : 2.0407
```

# An example

- Fitting the classification tree

```
library(tree)
pts.df <- cbind(pts.df, pres.abs$y)
colnames(pts.df)[8] <- "y"
pts.df$y <- as.factor(ifelse(pts.df$y == 1, "Yes", "No"))
tree.model <- tree(y ~ . , pts.df)
plot(tree.model)
text(tree.model, pretty=0)
```



# An example

- Fitting the classification tree

```r
summary(tree.model)
```

```
Classification tree:
tree(formula = y ~ ., data = pts.df)
Variables actually used in tree construction:
[1] "TempRange"        "PrecipWetQuarter" "ID"                             "MeanAnnTemp"
Number of terminal nodes:  8
Residual mean deviance:  0.3164 = 29.11 / 92
Misclassification error rate: 0.07 = 7 / 100
```

# Benefits and drawbacks

**Benefits**

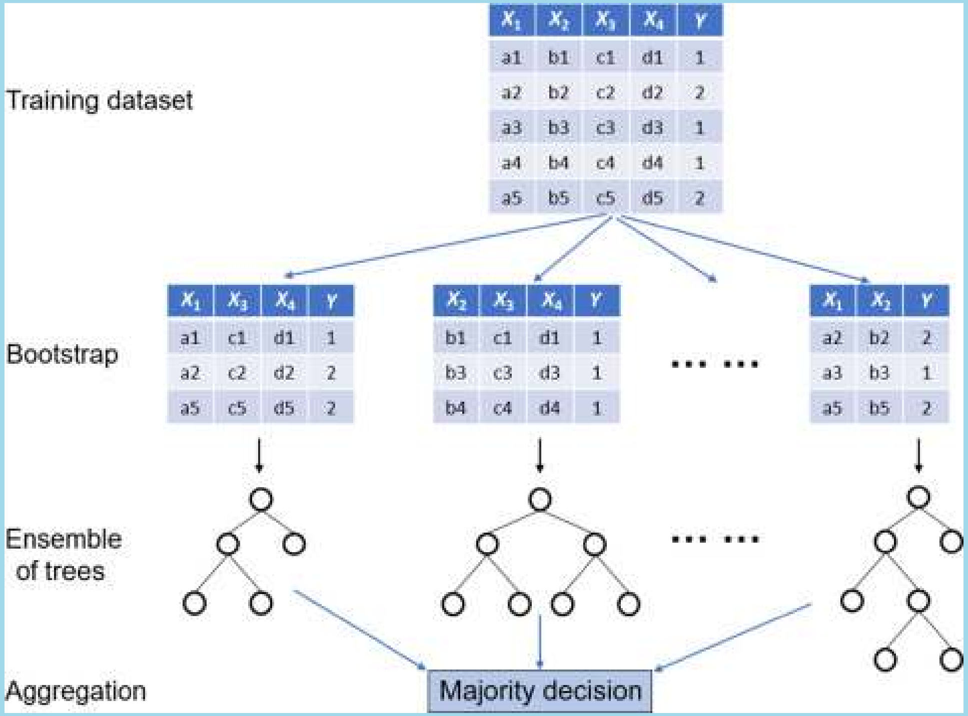
- Easy to explain
- Links to human decision-making
- Graphical displays
- Easy handling of qualitative predictors

**Costs**

- Lower predictive accuracy than other methods
- Not necessarily robust

# Random Forests

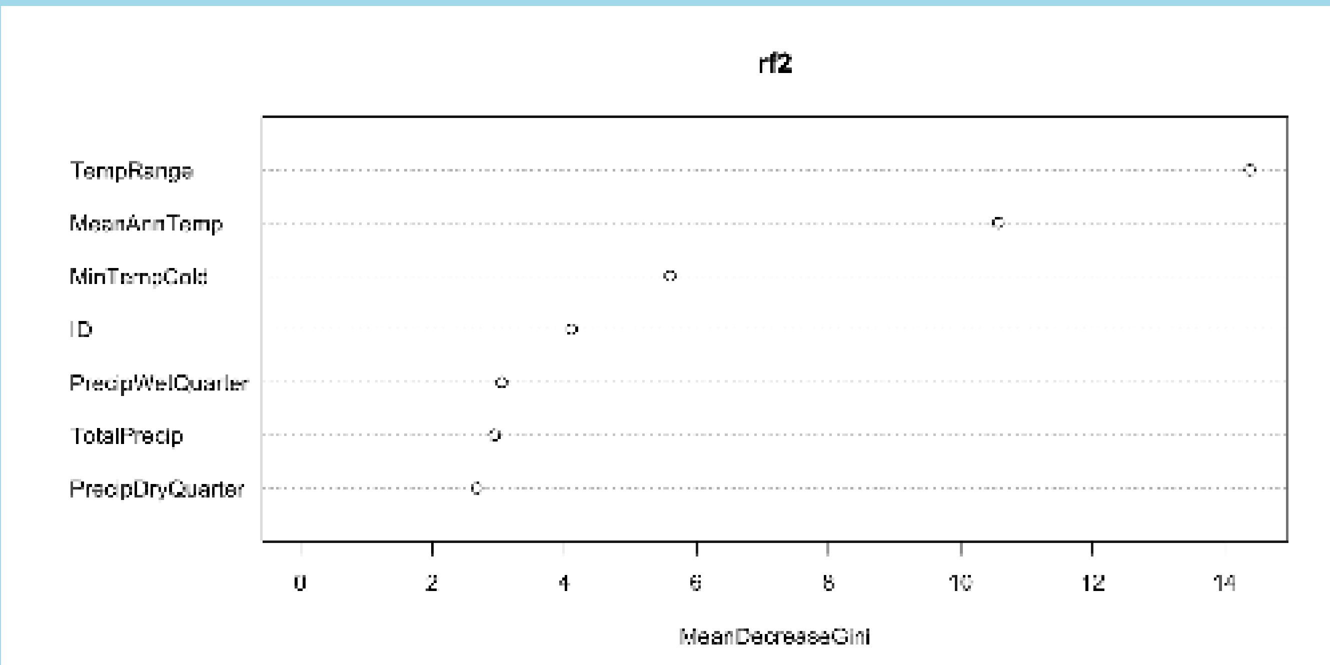
- Grow 100(000s) of trees using bootstrapping
- Random sample of predictors considered at each split
- Avoids correlation amongst multiple predictions
- Average of trees improves overall outcome (usually)
- Lots of extensions



# An example

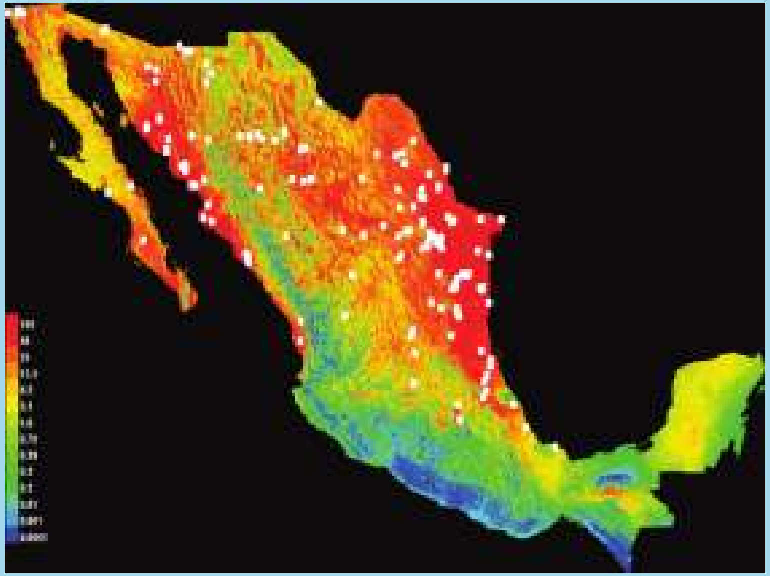
- Fitting the Random Forest

```r
library(randomForest)
class.model <- y ~ .
rf2 <- randomForest(class.model, data=pts.df)
varImpPlot(rf2)
```



# Modelling Presence-Background Data

# The sampling situation



From Lentz et al. 2008

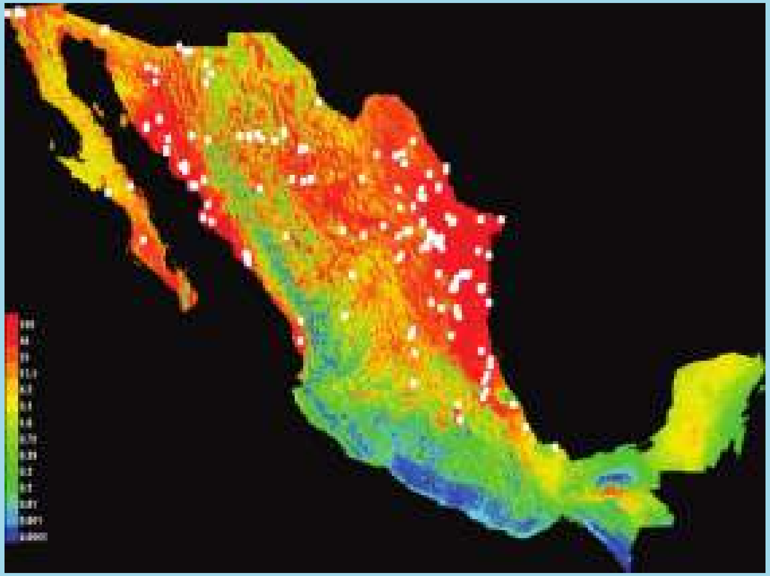
- Opportunistic collection of presences only
- Hypothesized predictors of occurrence are measured (or extracted) at each presence
- Background points (or pseudoabsences) generated for comparison

# The Challenge with Background Points

- What constitutes background?
- Not measuring *probability*, but relative likelihood of occurrence
- Sampling bias affects estimation
- The intercept

$$y_i \sim \mathrm{Bern}(p_i)$$
$$\mathrm{link}(p_i) = \mathbf{x_i}'\beta + \alpha$$

# MaxEnt



From Lentz et al. 2008

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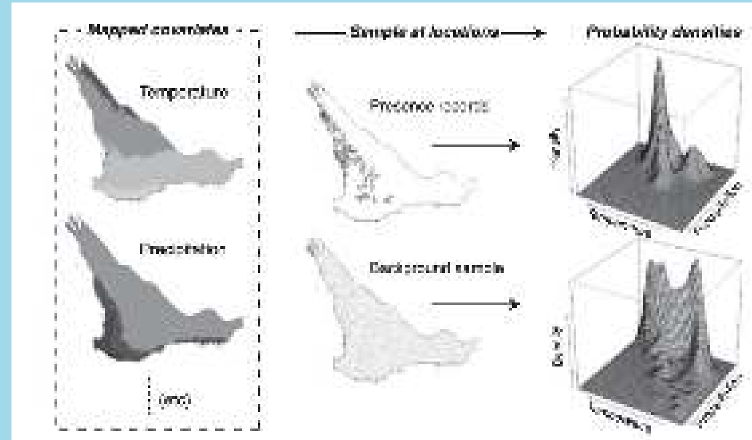
# Maximum Entropy models

- MaxEnt (after the original software)
- Need *plausible* background points across the remainder of the study area
- Iterative fitting to maximize the distance between predictions generated by a spatially uniform model
- Tuning parameters to account for differences in sampling effort, placement of background points, etc
- Development of the model beyond the scope of this course, but see Elith et al. 2010




From Elith et al. 2010

# Challenges with MaxEnt

- Not measuring *probability*, but relative likelihood of occurrence
- Sampling bias affects estimation (but can be mitigated using tuning parameters)
- Theoretical issues with background points and the intercept
- Recent developments relate MaxEnt (with cloglog links) to Inhomogenous Point Process models

# Extensions

- Polynomial, splines, piece-wise regression
- Neural nets, Support Vector Machines, many many more